Understanding and Using MINC

Background
In TOUGH2, the MINC (Multiple Interacting Continua) approach is used to model flow in fractured media. It is a generalization of the classical double-porosity concept developed by Warren and Root [Warren and Root, 1963]. As implemented in TOUGH2, selection of the MINC option activates a pre-processor that generates a secondary mesh used for the analysis. This secondary mesh is based on the original (primary) solution mesh and specified fracture parameters. The MINC approach provides several options to define the rock matrix/fracture flow connections and uses subgridding to resolve gradients in the matrix blocks.

The purpose of this document is to describe how to use the MINC option within PetraSim, a pre- and post-processor for TOUGH2. This description quotes liberally from the TOUGH2 user manual [Pruess, Oldenburg, and Moridis, 1999] and two reports by Karsten Pruess that provide detailed descriptions of MINC [Pruess, 1983], [Pruess, 1992]. This document was originally drafted by Shekhar Gosavi.

Figure 1 and Figure 2 illustrate the double porosity approach developed by Warren and Root. It is based on the notion that fractures have large permeability and small porosity (averaged over the reservoir), while the intact rock has the opposite characteristics. Therefore, any disturbance in reservoir conditions will travel rapidly through the network of interconnected fractures, while invading the matrix blocks only slowly. Based on this observation, the double-porosity concept assumes that global flow in the reservoir occurs only through the fracture system, which is described as an effective porous continuum. Rock matrix and fractures may exchange fluid (or heat) locally by means of “interporosity flow,” which is assumed to be “quasi-steady” and driven by the difference in pressures (or temperatures) between matrix and fractures.

![Figure 1: Idealized double porosity model of a fractured porous medium [Pruess, 1992]](image)
The crucial point in which MINC and conventional double-porosity methods differ is in the matrix-fracture exchange (the “interporosity flow”). The double porosity method assumes the interporosity flow is “quasi-steady,” which breaks down for non-isothermal and multi-phase flows. The MINC method treats interporosity flow in a fully transient way by computing the gradients which drive interporosity flow at the matrix-fracture interface. Matrix blocks are discretized into a sequence of nested volume elements, which are defined on the basis of distance from the fractures, Figure 3. Thus, interporosity flow is approximated as being one-dimensional.

Figure 3: Subgridding in the MINC method [Pruess, Oldenburg, and Moridis, 1999]

A schematic of a radial MINC model is shown in Figure 4.
The approach can also accommodate the more general “dual permeability” model, where flow occurs between both fracture and matrix blocks, Figure 5.

Figure 5: Flow connections in the "dual permeability" model. Global flow occurs between both fracture and matrix grid blocks. In addition there is fracture-matrix interporosity flow.
The MINC Processor

Input
Input to the MINC consists of:

- The primary mesh.
- The interporosity flow option (double-porosity, etc).
- The fracture orientations and corresponding fracture spacing.
- The number of nested interacting continua \( n \) and a set of corresponding volume fractions \( f_i \) where \( i < n \). The first volume fraction \( f_1 \) typically corresponds to the fracture. The unspecified final volume fraction will be calculated by MINC to preserve the total volume.

Proximity Function
For any given reservoir with a known fracture distribution, it is possible to determine the total matrix volume within a distance \( x \) from the fracture faces. This is illustrated in Figure 6.

![Figure 6: Illustration of partitioning matrix volumes within different distances from fractures](image)

The proximity function \( P(x) \) is used to calculate the total fraction of matrix volume within a distance \( x \) from the fracture faces. If the total volume of the domain is \( V_0 \), the total matrix volume is given by:

\[
V_m = (1 - f_1)V_0
\]

where \( f_1 \) is the fracture volume fraction. Then \( V(x) \), the matrix volume within a distance \( x \) from the fracture faces, is given by:

\[
V(x) = P(x)V_m
\]

A 1-D fracture spacing is shown in Figure 7.
The corresponding proximity function is given by:

\[ u = 2 \times \frac{x}{d_{frac}} \]

\[ P(x) = \begin{cases} 
  u, & 2x < d_{frac} \\
  1, & 2x \geq d_{frac}
\end{cases} \]

A 2D fracture system is shown in Figure 8.

The 2D proximity function is given by:

\[ u = 2 \times \frac{x}{d_{frac}} \]
\[ v = 2 \times \frac{x}{d_{frac}} \]

\[ P(x) = \begin{cases} 
  u + v - uv, & 2x < \min(d_{frac_x}, d_{frac_y}) \\
  1, & 2x \geq \min(d_{frac_x}, d_{frac_y})
\end{cases} \]
A 3D fracture system is shown in Figure 9.

![Figure 9: 3D fracture system](image)

The 3D proximity function is given by:

\[ u = 2 \frac{x}{dx_{frac}} \]
\[ v = 2 \frac{x}{dy_{frac}} \]
\[ w = 2 \frac{x}{dz_{frac}} \]

\[ P(x) = \begin{cases} 
  u + v + w - uv - vw - uvw, & 2x < \min(dx_{frac}, dy_{frac}, dz_{frac}) \\
  1, & 2x \geq \min(dx_{frac}, dy_{frac}, dz_{frac}) 
\end{cases} \]

Other, more general proximity functions are possible, but these are the ones supported by PetraSim.
**Calculation of Secondary Mesh**

Given the primary mesh, the fracture and volume fraction data, and the proximity function, MINC calculates the elements and connections for the secondary mesh.

**Element Data**

Each primary element is divided into *n* nested interacting continua. The first element is the fracture element, with the remaining elements used to calculate interporosity flow. The volume of each element is calculated using the volume fractions. The secondary mesh element records are written to the MINC file.

**Connection Data**

A new connection is created for each element nested in the original primary element. Thus, a connection is formed between secondary element 1 (the fracture element) and the adjacent matrix element 2, between (secondary) matrix elements 2 and 3, and so on. The connection records are written to the MINC file. For the double-porosity assumption, only the first fracture element connects to other fracture elements.

To calculate the connection data, we first need to calculate the distance *x*₂ within which each secondary matrix element lies, Figure 10. MINC calculates each *x*₂ based on the volume fraction and the proximity function and then calculates all the connection data using the derived information. The relation between the distance and the volume fraction is not based on the shape of the element, it is based on the proximity function.

![Figure 10: MINC partitioning of an idealized fracture system [Pruess, 1983]](image)

We know the total volume of the primary element *V₀*, the volume fraction of the fracture *f₁*, the volume fractions of the nested rock matrix elements *fᵢ* for 2 ≤ *i* ≤ *n*, and the proximity function *P(x)* for which
we are trying to find the distance $x_i$. Using the definition of the proximity function we can write the following equation for any nested element $i > 1$:

$$P(x_i) = \frac{V(x_i)}{V_m}$$

Using the volume fractions:

$$P(x_i) = \frac{\Sigma_{j=2}^{i} f_j V_0}{(1 - f_i)V_0}$$

or,

$$P(x_i) = \frac{\Sigma_{j=2}^{i} f_j}{(1 - f_i)}$$

This equation is solved using the bisection method to obtain $x_i$. This method is used for all nested elements except $x_1$, for which the distance from the fracture interface is taken to be zero.

At this point, we can calculate all $x_i$ shown in Figure 10. Note that the $x_i$ are independent of the mesh; they are a function only of the specified volume fractions.

We now position each node that represents the nested element at the midpoint between the calculated $x_i$. The last node is positioned at a point to give the most accurate estimate of the gradient at the interface of the inner element, Figure 11. Using these node positions we can specify the distances for the connectivity in each nested element.

<table>
<thead>
<tr>
<th>Case</th>
<th>Dimensions of matrix blocks</th>
<th>Dimensions of innermost blocks</th>
<th>Average linear dimension of innermost block</th>
<th>$D_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D</td>
<td>a</td>
<td>$u = a - 2x_{3-1}$</td>
<td>$l = u$</td>
<td>$l/6$</td>
</tr>
<tr>
<td>2-D</td>
<td>a</td>
<td>$u = a - 2x_{3-1}$</td>
<td>$l = \frac{2uv}{u+v}$</td>
<td>$l/8$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$v = b - 2x_{3-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D</td>
<td>a</td>
<td>$u = a - 2x_{3-1}$</td>
<td>$l = \frac{3uvw}{uvw + u + w}$</td>
<td>$l/10$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$v = b - 2x_{3-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>$w = c - 2x_{3-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 11: Quasi-steady flow distances for rectangular matrix blocks [Pruess, 1983]

Finally, the areas are evaluated at each $x_i$ by taking the derivatives of the volume:

$$A(x_i) = \frac{dV}{dx} = (1 - f_i)V_0 \frac{dP(x_i)}{dx}$$
Using MINC in PetraSim

The first step in implementing the MINC method is to create your model and mesh just as would be done for an unfractured porous medium. This “primary” mesh is then processed to generate a “secondary” mesh which includes the additional volume element and flow interface data needed to describe interporosity flow. This processing is performed automatically when the MINC option is selected.

To enable the MINC option in PetraSim, on the Properties menu click Global Properties, then click the MINC tab and select the MINC checkbox. Input the MINC parameters.

After MINC is selected, there will now be both matrix and fracture property tabs for each material. The fracture property is used by the first element in each group of the MINC secondary mesh and the matrix property is used by the nested elements.
Examples
The following examples illustrate the details of using MINC.

Single Cell 100m Reservoir with 50m Fracture Spacing
The first example is chosen so that the cell dimensions of the primary mesh are larger than the fracture spacing. The reservoir is 100m cube, represented by a single cell, Figure 12. The contents of the primary MESH file are shown in Figure 13. There is only one element with a volume of $1.0E6\,\text{m}^3$ and centered at 50m in the X, Y, and Z directions. Note: The MESH file has a fixed format, so a new number begins immediately after the exponent of the previous number. Alternate numbers have been shaded to help reading.

![Figure 12: Single cell model of 100m reservoir](image)

![Figure 13: Contents of the primary MESH file](image)

The MINC parameters are shown in Figure 14. We have selected the Double Porosity option with X, Y, Z – 3D fractures. The fracture spacing is 50m in all directions. The Number of Interacting Continua is 3 with specified volume fractions of 0.05, 0.2 (by specifying only 2 of the 3 volume fractions, TOUGH2 will calculate the last value so that the total is 1.0).

When TOUGH2 is run, the MINC processing is performed to create a MINC file that contains the secondary mesh. This MINC file is shown in Figure 15. There are now three elements, with the first number in their name indicating the subdivision. The first element represents the fracture and has a volume of $0.05E6\,\text{m}^3$ (this corresponds to the volume fraction times the total cell volume, $0.05\times1.0E6\,\text{m}^3$). The second and third elements are the nested matrix elements. Corresponding to the specified volume fractions, the volume of the second element is $0.20E6\,\text{m}^3$ and the volume of the third element is...
0.75E6m$^3$, which conserves the total volume of the original cell. All elements are centered at 50m in the X, Y, and Z directions.

There are two connections, the first is between element “1” and element “2 1” and the second connection is between element “2 1” and element “3 1”. To calculate $x_2$, we evaluate the desired value:

\[
P(x_2) = \sum_{j=2}^{3} \frac{f_j}{(1 - f_1)} = \frac{f_2}{(1 - f_1)} = \frac{0.20}{(1 - 0.05)} = 0.2105
\]

We then use the definition of the proximity function and iterate until we find the value of $x_2$ that gives us the value $P(x_2) = 0.2105$. This value is $x_2 = 1.894$. Thus, the first distance in the first connection is 0.0 and the second is 0.947m. Numerically evaluating the derivative at $x_2$ gives the area as 114000m$^3$.

The position of $x_3$ is found using the calculation in Figure 11. This is found to be $x_3 = 4.621$. Thus, the first distance in the second nested connection is 0.947m and the second distance is 4.621m. The area is determined to be 9.738E4m$^3$.

The demonstrates all the MINC calculations. A spreadsheet that implements these calculations is available at the PetraSim support site.

![Figure 14: MINC input parameters](image-url)
100m Reservoir with 50m Fracture and 25m Cell Spacing

In this example cell dimensions of the primary mesh are smaller than the fracture spacing. The reservoir is 100m cube, represented by 64 cells, Figure 16. The contents of the primary MESH file are shown in Figure 17. There are a total of 64 elements, each with a volume of 1.5620E4m$^3$.
All MINC parameters are the same as used in the single element example described above. We have selected the **Double Porosity** option with X, Y, Z – 3D fractures. The fracture spacing is 50m in all directions. The **Number of Interacting Continua** is 3 with specified volume fractions of 0.05, 0.2 (by specifying only 2 of the 3 volume fractions, TOUGH2 will calculate the last value so that the total is 1.0).

The secondary MINC file is shown in Figure 18. There are now a total of 192 elements. The first element represents the fracture and has a volume of 781.0m$^3$ (this corresponds to the volume fraction times the total cell volume, 0.05x1.5625E4m$^3$). The second and third elements are the nested matrix elements. Corresponding to the specified volume fractions, the volume of the second element is 3124.0m$^3$ and the volume of the third element is 11720m$^3$, which conserves the total volume of the original cell (within the accuracy of 5 significant figures).

There are now connections between the fracture elements and between the fracture and nested elements. The distances in the connections remain the same, but the areas change since the primary element volume is different.

<table>
<thead>
<tr>
<th>ELEM</th>
<th>1</th>
<th>3D.7810E+00 0.0000E+00 0.1250E+02 0.1250E+02 0.1250E+02</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3D.3124E+04 0.0000E+00 0.1250E+02 0.1250E+02 0.1250E+02</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3D.1172E+04 0.0000E+00 0.1250E+02 0.1250E+02 0.1250E+02</td>
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<table>
<thead>
<tr>
<th>CONNE</th>
<th>1</th>
<th>10.1250E+02 0.1250E+02 0.6250E+02 0.0000E+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>10.1250E+02 0.1250E+02 0.6250E+02 0.0000E+00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12</th>
<th>1</th>
<th>10.0000E+00 0.9471E+04 0.1781E+04</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>13</td>
<td>10.9471E+04 0.4621E+01 0.1521E+04</td>
</tr>
</tbody>
</table>

**Figure 18**: MINC secondary mesh for 25m cell size
Summary
We have reviewed the implementation on MINC and demonstrated its use in PetraSim. In the typical use of dual-porosity option, communication throughout the model is through the fractures and communication between the fractures and matrix is through the interporosity flow defined by nested elements. The proximity function implementation applies the volume fractions to the entire mesh. This means that the volume of the cells in the primary mesh is only used in calculated the nested volumes, but even a small volume primary cell will result in a cell with nested distances based on the fracture spacing, not the primary cell volume. We have provided numerical examples demonstrating the creation of the secondary mesh.

As described by Karsten Pruess [Pruess, 1983]:

“The MINC-method provides a rather substantial simplification of the complex problem of flow in a naturally fractured rock mass. It is not a patent recipe, but an approximation whose validity should be carefully evaluated before it is applied to specific problems. The concept of partitioning the rock matrix according to distance from the fractures is expected to be very accurate for certain systems and processes, while giving adequate engineering accuracy in others, but being poor or inapplicable in some areas.

- The MINC approximation is expected to be most accurate for flow systems with ubiquitous fractures and “small” matrix blocks, in which most blocks experience approximately uniform boundary conditions at all times.
- Generally favorable for application of the MINC method are single-phase flow problems, or problems with low matrix permeability, where interporosity flow is mostly heat conduction. In these cases gravity effects on interporosity flow will be either absent or small.
- Multiphase systems can be handled if matrix block dimensions are small in comparison to dimensions of regions with different phase compositions, or if density differences between the phases are “not too large”.
- Transport of chemical species in fractured rock masses should be amenable to a MNC representation, as species migration between matrix and fractures should be little affected by gravity. This will hold for chemical pollution in fissured systems, and for processes of ore formation in veins. Wall rock alterations in hydrothermal mineral systems are known to often depend primarily on the distance from the veins.
- The MINC—approximation is not applicable for systems with large matrix blocks which are subjected to non-uniform boundary conditions for extended time periods. This situation may arise in certain fractured petroleum reservoirs.”
Caution on Restart in PetraSim

In Version 4.2 of PetraSim, restart data for all nested cells is assigned the values of the first fracture cell. As a result, restart should only be used for continuing after the initial equilibrium state (where fracture and matrix cells will be in equilibrium). Restart should not be used during a transient analysis, since the transient data in the nested cells will be lost. This limitation is usually not a significant constraint to the analyst.
References


